

# Issues in Quantized Fractal Space Time

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## Abstract

In recent years, the picture of discrete space time has been studied in the context of stochastic theory. There are a number of ramifications, which are briefly examined. We argue that the causality of physics has its roots in the analyticity within the two dimensions of a fractal quantum path and further show how this picture has convergence with quantum superstrings.

## 1 Introduction

The concept of a discrete space time was formulated over fifty years ago by Schild, Snyder and others[1, 2]. It has survived over the years with variations through the works of Caldirola, Lee, Bombelli et al., Wolfe, the author and others ([3]-[11]). Infact, space time points imply infinite energies and momenta, but Quantum Theory has lived with this self contradiction[12]. Devices like renormalization have therefore to be invoked.

A second approach which avoids the difficulties of both the classical smooth and the discrete theory is that pioneered by Ord, Nottalee and El Naschie[54]-[56]. Those authors assume the existence of a transfinite Cantor-like space time Manifold  $\mathcal{E}^{(\infty)}$ .

This concept is also implicit in Dirac's relativistic theory of the electron [13]: Physically meaningful solutions arise only at and above the Compton scale. Below it we have the zitterbewegung effects.

Recently the author has pointed out that the electron can be fruitfully described with a Kerr-Newman metric, avoiding a naked singularity by invoking

a discrete space time structure, which can be further pushed forward in the context of a stochastic and fractal theory[14]-[18]. Such a formulation leads to a unification of electromagnetism and gravitation, and also shows up a cosmology consistent with the large number relations and recent observations, apart from providing a rationale for many other apparently ad hoc features. We will now briefly examine the above formulation and some of its ramifications.

## 2 The Kerr-Newman Formulation

It is well known that the Kerr-Newman metric describes the field of an electron including the anomalous gyro magnetic ratio  $g = 2$  [19] except that there is an inadmissible naked singularity: The radius of the horizon for the electron is given, in the usual notation by

$$r_+ = \frac{GM}{c^2} + ib, b \equiv \left( \frac{G^2 Q^2}{c^8} + a^2 - \frac{G^2 M^2}{c^4} \right)^{1/2} \quad (1)$$

It is to be noticed that the imaginary part of the radius in the above equation can be immediately identified with the imaginary part of the position operator of a Dirac electron, viz.,

$$x = (c^2 p_1 H^{-1} t + a_1) + \frac{i}{2} \hbar (\alpha_1 - c p_1 H^{-1}) H^{-1}, \quad (2)$$

The imaginary part in (26) gives the zitterbewegung and vanishes on averaging over Compton scales. It has been stressed[14, 15] that the Compton scale represents a minimum space time cut off, below which there is no meaningful physics. Physics emerges on averaging over these scales.

These minimum cut off scales have been shown to arise due to a stochastic underpinning [20, 21]. Indeed they give a meaning to Nelsonian and other stochastic studies [22]-[25]. All this is symptomatic of a background Zero Point Field or Quantum Vacuum which is the underpinning for the universe. The above Kerr-Newman formulation of the electron becomes more meaningful in a linearised General Relativistic context, where we have,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, h_{\mu\nu} = \int \frac{4T_{\mu\nu}(t - |\vec{x} - \vec{x}'|, \vec{x}')}{|\vec{x} - \vec{x}'|} d^3 x' \quad (3)$$

The justification for the linearized theory is that even at the Compton scale of the electron  $\sim 10^{-11}cms$  we are well outside the Schwarzschild radius of an electron  $\sim 10^{-56}cms$ [26] so that the linearized theory is applicable. Starting from (27) it has been shown that the correct spin, charge, gyro magnetic ratio, and infact the Kerr Newman metric itself can be obtained. This metric gives both the gravitational and the electromagnetic fields, and it has been shown that this is symptomatic of and leads to the desired unification of gravitation and electromagnetism. The formulation is on the face of it similar to Weyl's original theory, except that what is crucial is the purely Quantum Mechanical spinorial behaviour of the electron. Indeed the electromagnetic potential is given by

$$A^\mu = \hbar \Gamma_\sigma^{\mu\sigma} = \hbar \frac{\partial}{\partial x^\mu} \log(\sqrt{|g|})$$

which is apparently similar to the Weyl theory, except that this time the term is not put in ad hoc as in the Weyl formulation, but rather arises naturally due to the pseudo spinorial behaviour of the negative energy two component spinor  $\chi$  of the Dirac four spinor  $\begin{pmatrix} \Theta \\ \chi \end{pmatrix}$ . All this has been discussed in detail in the references[14, 15, 16].

Further at the Compton scale one gets a characterisation of the quark picture including such peculiar features as the fractional charge, handedness and confinement[18, 27, 20]. A crucial input here is the fact that while the usual three dimensionality arises well outside the Compton scale due to spin networks[19, 28], at the Compton scale itself we encounter lower dimensionality. Indeed, in the closely related non-Quantum Mechanical relativistic theory of particles [29], the centres of mass form a two dimensional disc within the analogue of the Compton scale.

We also get a characterisation of the neutrino and weak interaction[30]. Indeed in the above model the Compton wavelength of the neutrino becomes infinite as its mass vanishes and so the divide between the negative energy spinors and the positive energy spinors of the four spinor as in the case of the electron disappears, and along with it the double connectivity of space also. This immediately leads to the handedness of the neutrino [14]. Infact the neutrino is the divide between the Fermionic and the Bosonic particle.

### 3 Cosmology

It is easy to see how the Kerr-Newman type electrons, or more generally elementary particles can be formed out of a background Zero Point Field. As is known the energy of the fluctuations of the magnetic field in a region of length  $\lambda$  is[19]

$$B^2 \sim \frac{\hbar c}{\lambda^4} \quad (4)$$

where  $B$  to the left side of (28) denotes the magnitude of the magnetic field. If  $\lambda$  is the Compton wavelength, then the right side  $\sim mc^2$ , that is the whole particle can be created out of the background Zero Point Field. In what follows we take a pion to be a typical elementary particle, as in the literature, its mass being  $m$ . As there are  $N \sim 10^{80}$  particles in the universe, we should then have

$$Nm = M, \quad (5)$$

where  $M \sim 10^{56} gms$  is the mass of the universe. This is indeed so.

We can next deduce using the ZPF spectral density, the relation (cf.ref.[15]),  $M\alpha R$  where  $R$  is the radius of the universe. This is quite correct and in-fact poses a puzzle, as is well known and it is to resolve this dependence that dark matter has been postulated whereas in our formulation the correct mass radius dependence has emerged quite naturally. Other interesting and consistent consequences follow from the facts that the pion energy equals its gravitational energy and that  $\sqrt{N}$  particles are fluctuationally created in Compton time  $\tau$ :

$$\frac{GM}{c^2} = R, \sqrt{N} = \frac{2m_\pi c^2}{\hbar}.T \quad (6)$$

where  $T$  is the age of the universe  $\approx 10^{17} secs$ , and,

$$H = \frac{Gm_\pi^3 c}{\hbar^2}, \quad (7)$$

It is remarkable that equation (7) is known to be true from a purely empirical standpoint and is considered mysterious. Remembering that we are dealing with order of magnitude relations, we can deduce from (6) and (7) that

$$\frac{d^2 R}{dt^2} = \Lambda R \quad \text{where} \quad \Lambda \leq 0(H^2) \quad (8)$$

That is, a small cosmological constant cannot be ruled out. Equation (8) explains the puzzling fact that, if  $\Lambda$  exists, why is it so small[31]. To proceed we observe that the fluctuation of  $\sim \sqrt{N}$  in the number of particles leads to

$$\sqrt{N} = \frac{e^2}{Gm^2} \approx 10^{40},$$

whence we get,

$$R = \sqrt{N}l, \frac{Gm}{lc^2} = \frac{1}{\sqrt{N}}, G \propto T^{-1}, \quad (9)$$

as also the fact that  $\dot{G}/G \sim \frac{1}{T}$ , in reasonably good agreement [32].

Further, from the above we can deduce that the charge  $e$  is independent of time or  $N$ . Infact we can treat  $m$  (or  $l$ ),  $c$  and  $\hbar$  as the only microphysical constants and  $N$  as the only cosmological parameter, given which all other parameters and constants follow.

We can now easily deduce from (6), (7) and (8), the following:

$$\rho \propto T^{-1}, \Lambda \leq 0(T^{-2}), \quad (10)$$

In our model the equation (6) actually provides an arrow of time, in terms of the particle number  $N$ . Further, the cosmic background radiation can be explained in terms of fluctuations of interstellar Hydrogen[33]. In this model the universe continues to expand for ever with according to (10) decreasing density (unlike in the Steady State model). Indeed the fact that the universe would continue to expand for ever has since been observationally confirmed[34, 35]. Moreover, using (9) we can deduce the well known effects of General Relativity namely the precession of the perihelion of mercury, the gravitational bending of light, a recently observed anomalous inward acceleration in the solar system and the flattened galactic rotational curves without invoking dark matter[36, 30]. It may also be remarked that a background Zero Point Field does indeed show up as a cosmological constant, as has been shown in (Cf.ref.[30]).

## 4 Other Consequences

## 4.1 Weak Interactions

These can be characterised in terms of the semionic or anomalous statistics of the neutrino, as noted in Section 2. Using this we can deduce that

$$\frac{m_\nu c^2}{k} \approx \sqrt{3}T \quad (11)$$

At the present background temperature of about  $2^\circ K$ , this gives a neutrino mass

$$10^{-9}m_e \leq m_\nu \leq 10^{-8}m_e \quad (12)$$

where  $m_e$  is the electron rest mass. It is remarkable that (11) is exactly what is required to be deduced theoretically to justify recent models of lepton conservation or in certain unification schemes. We now observe that the balance of the gravitational force and the Fermi energy of the cold background neutrinos, gives[37]

$$\frac{GN_\nu m_\nu^2}{R} = \frac{N_\nu^{2/3} \hbar^2}{m_\nu R^2},$$

whence,  $N_\nu \sim 10^{90}$

where  $N_\nu$  is the number of neutrinos, as indeed is known.

If the new weak force is mediated by an intermediate particle of mass  $M$  and Compton wavelength  $L$ , we will get from the fluctuation of the particle number  $N_\nu$ , on using (12),

$$g^2 \sqrt{N_\nu} L^2 \approx m_\nu c^2 \sim 10^{-14}, \quad (13)$$

From (13), on using the value of  $N_\nu$ , we get,  $g^2 L^2 \sim 10^{-59}$

This agrees with experiment and the theory of massless particles the neutrino specifically acquiring mass due to interaction[38], using the usual value of  $M \sim 100 \text{Gev}$ .

Additionally there could be a long range force also, a "weak electromagnetism" with coupling  $\bar{g}$ . This time, in place of (13), we would have,

$$\frac{\bar{g}^2 \sqrt{N_\nu}}{R} \approx 10^{-8} m_\nu c^2 \quad (14)$$

Comparing (14) with a similar equation for the electron, we get[30],

$$\bar{g}^2 / e^2 \sim 10^{-13}$$

so that in effect the neutrino will appear with an "electric charge" a little less than a millionth that of the electron.

Interestingly from an alternative perspective [39], it can be concluded that the cosmological Neutrino background can mediate long range forces  $\sim \frac{1}{r^2}$  for  $r \ll \frac{1}{T}$  and  $\sim \frac{1}{r^6}$  for  $r \gg \frac{1}{T}$ .

In any case using the value of the Neutrino "electrical" charge and treating it like a Kerr-Newman black hole, as in the case of the electron it is easy to deduce a magnetic moment for it:

We start with the relation[15]

$$\hbar \sim \frac{Gm^2\sqrt{N}}{c}$$

, where  $m$  is the electron mass. For the neutrino mass as given in (12) and particle number  $N_\nu$ , the above becomes,

$$\hbar' \sim 10^{-12}\hbar,$$

which is symptomatic of the bosonic (or spin zero) behaviour of the neutrino. Using this value  $\hbar'$  instead of  $\hbar$ , and the neutrino "electric" charge and mass as given above, we can deduce that its magnetic moment is given by

$$\mu_\nu \sim 10^{-11} \text{ Bohr magnetons}$$

Indeed, this is in agreement with known values [40].

## 4.2 Discrete Space-time Effects

Within the above scheme, the neutral pion has been exhibited as a bound state of an electron and a positron. At first sight one would expect that such a bound state would cause pair annihilation and would lead to the appearance of two photons. However the existence of such a bound state is an imprint of discrete space time as can be seen by the following argument: In this case the the Schrodinger equation is given by (Cf. also [4])

$$H\phi T = E\phi T = \phi i\hbar \left[ \frac{T(t + \tau/2) - T(t - \tau/2)}{\tau} \right] \quad (15)$$

where  $\phi$  is the space part and  $T$  is the time part of the wave function and  $\tau$  is the minimum unit, the Compton time. From (15) one can deduce that

the most excited stable state appears at the critical value

$$E \sim \frac{\hbar}{\tau} \sim mc^2$$

which is ofcourse the pion energy. It must be pointed out that the decay mode of the pion bears out these arguments.

Thus the existence of the pion as a bound state of an electron and a poistron due to discrete space time is similar to the original Bohr orbits, at the birth of Quantum Mechanics, as will be seen below.

In the same vein, it was argued that the Kaon decay puzzle, wherein time reversibility is violated, could be explained on similar lines[41]. Let us now consider the effect more closely. Indeed it has been shown that the discreteness leads to a non commutative geometry [20] viz.,

$$[x, y] = 0(l^2), [x, p_x] = i\hbar[1 - l^2] \quad (16)$$

and similar equations. If terms  $\sim l^2$  are neglected we get back the usual Quantum Theory. However if we retain these terms, then we can deduce the Dirac equation. Moreover it can be seen that given (16) space reflection symmetry no longer holds. This violation is an  $O(l^2)$  effect.

This is not surprising. It has already been pointed out [42] that the space time divide viz.,  $x + \imath ct$  arises due to the zitterbewegung or double Weiner process in the Compton wavelength - and in this derivation terms  $\sim (ct)^2 \sim l^2$  were neglected. However if these terms are retained, then we get a correction to the usual theory including special relativity.

To see this more clearly let us (Cf.ref.[14, 43]) as a first approximation treat the continuum as a series of discrete points separated by a distance  $l$ , which then leads to

$$Ea(x_n) = E_0a(x_n) - Aa(x_n + l) - Aa(x_n - l) \quad (17)$$

When  $l$  is made to tend to zero, it was shown that from (17) we recover the Schrodinger equation, and further, we have,

$$E = E_0 - 2Acoskl. \quad (18)$$

The zero of energy was chosen such that  $E = 2A = mc^2$ , the rest energy of the particle (Cf.[14]), in the limit  $l \rightarrow 0$ . However if we retain terms  $\sim l^2$ ,



then from (18) we will have instead

$$\left| \frac{E}{mc^2} - 1 \right| \sim 0(l^2)$$

The above shows the correction to the energy mass formula, where again we recover the usual formula in the limit  $O(l^2) \approx 0$ .

It must be mentioned that all this would be true in principle for discrete space time, even if the minimum cut off was not at the Compton scale.

Intuitively this should be obvious: Space time reflection symmetries are based on a space time continuum picture.

### 4.3 A Mass Spectrum

It is possible to obtain the masses of different elementary particles by considering them to be suitable bound states of the leptons, on the basis of their decay modes, in the context of the above picture[44].

For example a proton could be considered to be a bound state of two positrons and a central electron[17, 18]. In this case we recover the correct mass of the proton,  $m_P$ . There is also a string of excited states with masses  $(2n + 1)m_P$ . It is quite remarkable that the  $\Omega_c$  baryon has a mass nearly  $3m_P$ , after which there is a big gap and the next baryon viz.,  $\Lambda_v$  has a mass  $5m_P$ . Similarly other shortlived baryons whose masses are odd multiples of the proton masses can be expected.

Taking into account the rotational degrees of freedom, it is possible to get an additional mass spectrum(Cf.ref.[44]) viz.,

$$mc^2 = \frac{2(n + 1/2)\hbar\omega}{1 - \frac{5}{6}k^2},$$

where for  $n = 0$  and  $k = 0$ ,  $\hbar\omega = m_P c^2$ . For  $k = 1$  this then gives a mass six times that of a proton and so on we can generate a series of masses.

### 4.4 The Magnetic Effects

If the electron is indeed a Kerr-Newman type charged black hole, it can be approximated by a solenoid and we could expect an Aharonov-Bohm type

of effect, due to the vector potential  $\vec{A}$  which would give rise to shift in the phase in a two slit experiment for example [45]. This shift is given by

$$\Delta\delta_{\vec{B}} = \frac{e}{\hbar} \oint \vec{A} \cdot d\vec{s} \quad (19)$$

while the shift due to the electric charge would be

$$\Delta\delta_{\vec{E}} = -\frac{e}{\hbar} \int A_0 dt \quad (20)$$

where  $A_0$  is the electrostatic potential. In the above formulation (Cf.ref.[46], we would have

$$\vec{A} \sim \frac{1}{c} A_0 \quad (21)$$

Substitution of (21) in (19) and (20) shows that the magnetic effect  $\sim \frac{v}{c}$  times the electric effect.

Further, the magnetic component of a Kerr-Newman black hole, as is well known [19] is given by

$$B_{\hat{r}} = \frac{2ea}{r^3} \cos\Theta + 0(\frac{1}{r^4}), B_{\hat{\Theta}} = \frac{e a \sin\Theta}{r^3} + 0(\frac{1}{r^4}), B_{\hat{\phi}} = 0, \quad (22)$$

while the electrical part is

$$E_{\hat{r}} = \frac{e}{r^2} + 0(\frac{1}{r^3}), E_{\hat{\Theta}} = 0(\frac{1}{r^4}), E_{\hat{\phi}} = 0, \quad (23)$$

Equations (22) and (23) show that in addition to the usual dipole magnetic field, there is a shorter range magnetic field given by terms  $\sim \frac{1}{r^4}$ . In this context it is interesting to note that an extra  $B^{(3)}$  magnetic field of shorter range and mediated by massive photons has indeed been observed and studied over the past few years[47].

## 4.5 Unification of Fluctuations

It is quite remarkable that in the preceding considerations, we get the gravitational and electromagnetic interactions as also the weak interaction from fluctuation in the particle number  $\sim \sqrt{N_\nu}$  or  $\sqrt{N}$ , for example in the equations leading to (9) or (13). It is thus the fluctuation or what has been characterised as non local effects that underlie all the interactions[48] and lead to a unified picture. It is interesting that El Naschie's fluctuon is very much in this spirit[49].

## 4.6 Fractal Matter

It has been remarked earlier that as we approach the Compton wavelength, there is a change in dimensionality - we go to two and one dimensions. Such a low dimensional behaviour leads to fractional charges and superconductivity type effects[50, 51, 52]. It is indeed pleasing that experiments with carbon nano tubes already indicate such phenomenon. This type of behaviour should be exhibited by quarks also.

## 4.7 Miscellaneous Matters

a) The discrete space time or zitterbewegung has an underpinning that is stochastic. The picture leads to the goal of Wheeler's 'law without law' [53, 20, 21]. Furthermore the picture that emerges is machian. This is evident from equations like (6), (7) and (9)– the micro depends on the macro. So the final picture that emerges is one of stochastic holism.

b) Another way of expressing the above point is by observing that the interactions are relational. For example, in the equation leading to (9), if the number of particles in the universe tends to 1, then the gravitational and electromagnetic interactions would be equal, this happening at the Planck scale, where the Compton wavelength equals the Schwarzschild radius[54, 55].

c) Infact as shown [42], when  $N$  the number of particles in the universe is 1 we have a Planck particle with a short life time  $\sim 10^{-42}secs$  due to the Hawking radiation but with  $N \sim 10^{80}$  particles as in the present universe we have the pion as the typical particle with a stable life time  $\sim$  of the age of the universe due to the Hagedorn on radiation.

d) It is well known that there are 18 arbitrary parameters in contemporary physics. We on the other hand have been working with the micro physical constants referred to earlier viz., the electron (or pion) mass or Compton wavelength, the Planck constant, the fundamental unit of charge and the velocity of light. These along with the number of particles  $N$  as the only free parameter can generate the mass, radius and age of the universe as also the Hubble constant.

If we closely look at the equation leading to (9) giving the gravitational and electromagnetic strength ratios, we can actually deduce the relation,

$$l = \frac{e^2}{mc^2} \quad (24)$$

In other words we have deduced the pion mass in terms of the electron mass, or, given the pion mass and the electron mass, we have deduced the fine structure constant. From the point of view of the order of magnitude theory in which the distinction between the electron, pion and proton gets blurred, what equation (24) means is, that the Planck constant itself depends on  $e$  and  $c$  (and  $m$ ). Further in the Kerr-Newman type characterisation of the electron, the charge  $e$  is really equivalent to the spinorial tensor density ( $N = 1$ ), Cf.ref.[14]. In this sense  $e$  also is pre determined and we are left with a minimum length viz. the Compton length and a minimum time viz. the Compton time (or a maximal velocity  $c$ ) as the only fundamental constants.

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## APPENDIX

### A Brief Note on Analyticity and Causality, and the "Levels of Physics"

In previous communications[1, 2] it was argued that within the Compton wavelength of a fractal Brownian path the one dimensional coordinate  $x$  becomes complex. That is  $x$  becomes  $x + ix'$  where, further it was shown that  $x' = ct$ , and it was argued that this is the origin of Special Relativity. It is relevant to mention that complex time has also been considered by El Naschie[3, 4].

Within the Compton scale we have a Hawking-Hartle[5] situation, where time becomes imaginary and static - in these regions space time can be represented by the compact rotation group. This is also from the point of view of ordinary space-time, the unphysical zitterbewegung region, and as Dirac[6] pointed out, physics begins after an integration over this region. Outside the Compton scale however, which is our physical domain[7], we have the Minkowski metric.

Interestingly it was argued[1] that the above consideration was also at the root of the complex wave function of Quantum Mechanics, which differentiates it from classical physics. Infact if we take the Quantum Mechanical wave function to be the carrier of as much physical information of the state as is possible then we can argue that because the first time derivative of the wave function is then not required, unlike in classical theory, where position and velocity are independantly required, it is necessary that the wave function be complex in order to preserve causality[8]. Indeed if the wave function were real, we would have a stationary picture with a constant probability current. Within the Compton scale, that is in the domain of the Hawking-Hartle static time  $t'$ , we have

$$x^2 + y^2 + z^2 + c^2 + t'^2 = \text{invariant} \quad (25)$$

Further, analyticity demands the Cauchy-Reimann equations which lead to, in this case, the Laplacian operator equation,

$$\left[ \nabla^2 + \frac{1}{c^2} \frac{\partial^2}{\partial t'^2} \right] \psi = 0 \quad (26)$$

However, when we cross over to the domain of our usual physics, firstly  $t'$  goes over to  $it$ , where  $t$  is our physical time. Secondly the region of analyticity



in our physical world is the region outside the Compton wavelength, and excludes the region within which is the unphysical zitterbewegung region of non local or superluminal effects (Cf.[6]). So as Dirac pointed out, after the integration over the unphysical Compton scale, and remembering that,

$$\psi = \sqrt{\rho} e^{iS},$$

where  $\rho$  is the probability density of  $x(t)$ , so that an integration of  $\psi\psi^*$  over the Compton region gives the mass, we get, instead of (25) and (26),

$$x^2 + y^2 + z^2 - c^2 t^2 = \text{invariant} \quad (27)$$

$$\square\psi = mc^2\psi \quad (28)$$

Equation (27) is the Minkowski metric while equation (28) can be easily recognised as the Klein-Gordon equation.

There is another way to derive the D'Alembertian in (28). For this we use (27), along with the fact that, precisely due to the double Weiner process at the Compton scale, referred to above, as pointed out by Nottale[9] the energy momentum operators are given by

$$\left( \frac{\hbar}{i} \vec{\nabla}, \frac{\hbar}{i} \frac{\partial}{\partial t} \right)$$

Thus causality in the physical world as expressed by (27) or (28) is related to analyticity within the unphysical Compton region.

We next observe that as above, the discretization at the Compton scale leads to the commutation relations[10, 11]:

$$[x, p_x] = i\hbar[1 - l^2] \quad (29)$$

Equation (29) implies a correction to the usual uncertainty relation, due to the minimum space-time cut off. This is exactly the case in Quantum superstrings[12, 13]. There also, due to duality, a minimum cut off emerges and we have the equation (29).

What we would like to point out is that we are seeing here different levels of physics. Indeed, rewriting (29) as,

$$[x, u_x] = i[l - l^3],$$

we can see that if  $l = 0$ , we have classical physics, while if  $0(l^3) = 0$ , we have Quantum Mechanics and finally if  $0(l^3) \neq 0$  we have the above discussed fractal picture, and from another point of view, the superstring picture. Interestingly, in our case the electron Compton wavelength  $l \sim 10^{11}cm$ , so that  $0(l^3) \sim 10^{-33}$  as in string theory.

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